Math 250A: Reading and Concepts for 9/14-9/24

General reading note: This marks the beginning of a new section in our course: the study of rings. In the book this corresponds to most of Chapter 2 and a little of Chapter 4 (for the most part, Chapter 4 will be a part of the next section in the course, i.e. Galois theory). I will assume from the start that you are comfortable with basic definitions of rings, ideals, quotient rings, field of fractions (or field of quotients) and the like. Some other remarks: Lang calls integral domains "entire rings" most of the time; now that Lang has covered category theory in section 1.11, he sometimes states some facts about rings in category-theoretical language – if you are not comfortable with it, don't worry about it yet; when we get to polynomial rings in this section of the course, I will assume you are fairly familiar with everything discussed about polynomial rings in Section 2.3 (don't worry about group rings).

The lectures this week will be planned roughly as follows:

- 9/14: Overview of what we will cover in the rings section, set of ideals of a ring as an additive/multiplicative monoid, ring homomorphisms, ring isomorphism theorems, beginning Chinese Remainder Theorem for rings. Reading: Section 2.1, CRT part of Section 2.2. I assume you know what the following are: rings, commutative rings, integral domains, fields, unity, units, ideals, quotient rings, principal ideals.
- 9/17: Chinese remainder theorem continued, applications thereof. Beginning the proof that every PID (which Lang calls a principal entire ring) is a UFD (which Lang calls a factorial ring). Reading: Section 2.2, beginning Section 2.5. I will assume you know what a PID (principal ideal domain) is and what a UFD (unique factorization domain) is. I also assume you are comfortable with prime ideals and irreducible elements.
- 9/19: Continuing (finishing?) discussion of PID's versus UFD's. Reading: Section 2.5. Concepts you should know: same as last time.
- 9/21: Showing that if D is a UFD, so is $D[x]$. Reading: Section 4.2. I assume you are fairly comfortable with the polynomial ring $R[x]$ where R is a ring: to refresh your memory of these rings, you can read Section 2.3.
- 9/24: Hilbert's theorem. Reading: Section 4.4. For this lecture it will be helpful for you to recall (from previous lectures) what a PID and a Noetherian ring is.