Math 250A: Reading and Concepts for 9/5-9/12

The lectures this week will be planned roughly as follows:

- 9/5: Finishing off group actions, beginning automorphism groups. Reading: Section 1.5. There isn’t much in Lang on automorphism groups and semidirect products. I assume you know what a direct product, center, and centralizer is, as well as what is meant by conjugate elements and groups. It might be helpful for you to be familiar with the dihedral groups $D_n$ and their center, as well as to think about what the center of $S_n$ is for $n \geq 3$.
- 9/7: Semidirect products, Recognition theorem, and applications thereof. Reading: this isn’t really contained in Lang but will come in handy when we go on to applications of Sylow theorems. You should be comfortable with automorphism groups, and the class equation.
- 9/10: Sylow theorems. Reading: Section 1.6. I will assume you have read and understood the proof of Lemma 6.1 in the book, and that you are comfortable with the results of Section 1.5 in the context of the conjugation action.
- 9/12: Sylow theorems continued, some applications. Reading: Section 1.6. It will help to review our discussion of semidirect products and automorphism groups for the purpose of the applications. Note: if we get to this point today, we will say little about the sections on finitely generated abelian groups and free groups other than to point out some of the main results/notions. The proofs of the main theorems here, as well as the terms introduced are better covered once we study modules over a PID later in the course, as the results in the context of abelian groups are a direct consequence of the more general results there (see Section 3.7, which Lang starts by saying that he is about to generalize the results of Chapter 1 and that the proofs are essentially the same). We will also delay our introduction to category theory (section 1.11) until we study modules, as it is very naturally introduced at that stage. Do not despair if you were really itching to get to that part!