Math 250A: Reading and Concepts for 8/24-8/31

General reading note: If you are not comfortable with the material covered in Section 1.2 (groups) and 1.4 (cyclic groups) of Lang, please read it before we begin. While you need not understand *all* of the examples there, you should be fluent in the definitions and the theorems proven. Note that Lang calls what is often called the *order* of an element in G the *period* of the element in Section 1.4. Also, for a summary of things you need to know about cyclic groups, see Arthur Ogus's nice concise notes on our course website. To refresh your comfort with permutation groups, it might help also to look over George Bergman's notes on the proof that A_n is simple for $n \geq 5$ (also posted on the course website).

The lectures this week will be planned roughly as follows (it may very well take longer to cover than planned):

- 8/24: Isomorphism theorems, exact sequences, begin composition series (what Lang refers to as normal towers in which the quotients are simple). Reading: Section 1.3. I assume you know what the following are: groups, (normal) subgroups, quotient groups, index, homomorphism, image, kernel, the canonical morphism $G \to G/H$ where $H \trianglelefteq G$, centralizer and normalizer. It will be useful to look over the italicized exercises on page 15 and convince yourself they are true.

- 8/27: Composition series continued, Jordan-Hölder theorem, abelian and cyclic towers. Reading: Section 1.3. I assume you know what simple, abelian, and cyclic groups are. You should be comfortable with the isomorphism theorems covered last time, and should convince yourself of the following: If G_1, H_1 are normal subgroups of G, then $G_1 H_1$ is a normal subgroup of G. Also, if H is a normal subgroup of G and K is a normal subgroup of G contained in H, then H/K is a normal subgroup of G/K. Finally, the intersection of normal subgroups is normal.

- 8/29: Solvable groups. Reading: Section 1.3. Same suggestions on what you should know and be comfortable with as last time.

- 8/31: Review of group actions and class equation; automorphism groups, possibly starting semidirect products and applications thereof. Reading: Section 1.5. There isn't much in Lang on automorphism groups and semidirect products. I assume you know what a direct product, center, and centralizer is, as well as what is meant by conjugate elements and groups. It might be helpful for you to be familiar with the dihedral groups D_n and their center, as well as to think about what the center of S_n is for $n \geq 3$.