1. Let $M$ be an $R$-module, and suppose there is a surjective $R$-linear homomorphism $\phi : M \to R$. Show that $M \cong \ker(\phi) \oplus R$.

2. Let $M$ be an $R$ module and let $N$ be a submodule of $M$. Prove that if $M/N$ and $N$ are finitely generated then $M$ is finitely generated.

3. Recall that a module $M$ is called irreducible if the only submodules it has are $M$ and $0$.
   
   (a) Prove that if $R$ is commutative, then $M$ is irreducible iff $M \cong R/I$ as $R$-modules for some maximal ideal $I$ of $R$.
   
   (b) If $M$ and $N$ are irreducible $R$-modules, prove that any nonzero $R$-module homomorphism from $M$ to $N$ is an isomorphism.

4. Prove that an $R$-module $N$ satisfies the ACC for submodules iff there is in every nonempty set $U$ of submodules of $N$ a maximal element $M$ (here maximal means $M$ is not properly contained in any other element of $U$).

5. Let $C$ be a cyclic $D$-module of order $\mu$, where $D$ is a PID.
   
   (a) Prove that every submodule of $C$ is cyclic with order a divisor of $\mu$.
   
   (b) For each principal ideal $(\lambda)$ of $D$ with $(\lambda) \supset (\mu)$, show that $C$ has exactly one submodule which is cyclic of order $\lambda$.

6. If $D = F[x]$, with $F$ a field, show that a cyclic $D$-module of order $f(x) \in F[x]$ is also a vector space over $F$ of dimension the degree of the polynomial of $f$. 