Math 250A Homework 1, due 8/31/2012

1) Do problems 3, 4 (no need for the extra assumption on K in this problem: you can drop it and interpret HK as merely a set), 9, 10, 13, 14 from chapter I in the book.

2) A subgroup H of a group G is called *characteristic* if every automorphism of G maps H to itself. Let $G^{(0)} = G$ and let $G^{(i)} = [G^{(i-1)}, G^{(i-1)}]$ be the commutator subgroup of $G^{(i-1)}$ for $i \geq 1$. Show that $G^{(i)}$ is a characteristic subgroup of G for all $i \geq 0$.

3) a) Show that an abelian group has a composition series if and only if it is finite.

b) Let F be a field and let $\mathrm{GL}_n(F)$ denote the group of $n \times n$ invertible matrices with entries in F (the group operation is matrix multiplication). Show that $\mathrm{GL}_n(F)$ has a composition series if and only if F is finite.