Math 113: What we covered from Sections 45, 46, and 47

As promised, here is a fairly detailed account of what we covered in class from sections 45, 46, and 47.

Section 45: Everything in pp. 389-393, Theorem 27.25 as a corollary of Lemma 45.12, Theorem 45.17 (statement only, though we proved the two theorems leading up to it which make the proof of 45.17 quite straightforward, so it may be a good exercise to work it out for yourself), Theorem 45.29 (statement only), Example 45.31 and comment right above it (that not every UFD is a PID). We also mentioned that $\mathbb{Z}[\sqrt{-5}]$ is not a UFD, which is a fact you should know. For details, see Example 47.9. Do keep in mind that, while we didn’t discuss most of pp. 396-399 in full generality, we did show all of it in the special case where $D = \mathbb{Z}$ when we worked through Section 23 of the book (just before spring break), so you are expected to be familiar with it in that context. Around that time, we also defined what a greatest common divisor of two polynomials in $F[x]$ is, and you should know it: the more general definition is given in Definition 45.19, if you’re interested.

Section 46: pp. 401-403 minus the proof of Theorem 46.6. Again, even though we didn’t discuss the Euclidean algorithm for general Euclidean domains (pp. 404-405), we did discuss it in the special case of $F[x]$ in order to show unique factorization in $F[x]$ right after spring break, so that is something you should be familiar with.

Section 47: pp. 407-409, Definition 47.6, Theorem 47.7 (statement only), Theorem 47.10 (with proof).