Math 113 Homework 7, due 3/16/2012


2) Let $C$ denote the set of colors red, black, blue. Recall that any subgroup $G$ of $S_n$ acts on the set $X' = \{1, 2, \ldots, n\}$. Let $C^n$ denote the set of $n$-tuples of colors from $C$ (for example $(c_1, c_2, \ldots, c_n) = (\text{red}, \text{red}, \text{black}, \text{blue}, \text{red}, \ldots) \in C^n$). We can think of $C^n$ as the set of colorings of $X$ in red, black, and blue in some sense.

a) Show that the map $G \times C^n \to C^n$ defined by

$$\sigma(c_1, c_2, \ldots, c_n) = (c_{\sigma(1)}, c_{\sigma(2)}, \ldots, c_{\sigma(n)})$$

is a group action.

b) For any permutation $\sigma \in G$, let $r(\sigma)$ denote the number of cycles in the disjoint cycle decomposition of $\sigma$ where we do count cycles with just one element. If we denote $C_n$ by $X$, show that

$$|X_\sigma| = 3^{r(\sigma)}$$

for any $\sigma \in G$.

c) What does Burnside’s Lemma tell us about the number of orbits of $A_3$ acting on $C_n$?

3) How challenging was this homework? How long did it take?