1) From the book: do problems 5.12, 5.13, 5.43, 5.54, 6.32. In problem 6.32, make sure to give a brief justification of your answer. It is good practice for exams, since you will have some T/F questions on every test.

2) Let $n \geq 3$ be an integer and let $\theta = 2\pi/n$. Let $P_n$ be the regular $n$-gon with vertices $(\cos j\theta, \sin j\theta)$ for $j \in \mathbb{Z}_n$. In class we learned that the dihedral group $D_n$ is the symmetry group of $P_n$, which consists of rotations $\rho_j$ and reflections $\mu_j$ for $j \in \mathbb{Z}_n$. For this exercise $\rho_j$ is taken to be the counterclockwise rotation around the origin by angle $j\theta$, and $\mu_j$ is the reflection across the line through the origin and the point $(\cos(j\theta/2), \sin(j\theta/2))$.

Find (and give at least some justification for) general formulas for $\rho_i \rho_j$, $\rho_i \mu_j$, $\mu_i \rho_j$, and $\mu_i \mu_j$ where the composition of motions $\sigma \tau$ is just $\tau$ followed by $\sigma$. For example, $\rho_i \rho_j = \rho_{i+j}$, where the addition of indices is mod $n$. You don’t need to reprove that $D_n$ is a group. (*Hint: One distinction between a rotation and a reflection in this case is that reflection fixes the line through which one reflects, and rotation fixes no line unless it is rotation by $2\pi$.)

3) a) Find all subgroups of the dihedral group $D_4$ (symmetries of the square).

b) Recall that the center $Z(G)$ of a group $G$ is the set

$$Z(G) = \{x \in G \mid xy = yx \text{ for all } y \in G\},$$

and that it is a subgroup of $G$. Find the center of $D_n$ for $n \geq 3$. (*Hint: use the formulas you found in problem 2. The answer depends on whether $n$ is even or odd.)*

4) How hard was this homework for you? How long did it take?