Math 113 Homework 2, due 2/3/2012

1) Do problems 3.11, 3.15, 3.24, 3.33, 4.15, 4.21, 4.32, 4.38 from the book.

2) Let $n > 1$ be an integer and let $\mathbb{Z}_n^*$ be the set of units in $\mathbb{Z}_n$, i.e. the elements $x \in \mathbb{Z}_n$ for which there exists $y \in \mathbb{Z}_n$ with $xy = 1$.
   
   a) Show that $\mathbb{Z}_n^*$ with multiplication is a group.
   
   b) Make multiplication tables for $\mathbb{Z}_8^*$, $\mathbb{Z}_{10}^*$, and $\mathbb{Z}_{12}^*$.
   
   c) Show that $\mathbb{Z}_8^* \approx \mathbb{Z}_{12}^*$ but $\mathbb{Z}_8^* \not\approx \mathbb{Z}_{10}^*$ and $\mathbb{Z}_{10}^* \not\approx \mathbb{Z}_{12}^*$.

3) Let $S_3$ denote the group of permutations of 3 letters (where the binary operation is composition). Let $x$ denote the cyclic permutation which sends $(1, 2, 3)$ to $(2, 3, 1)$, and let $y$ denote the permutation which sends $(1, 2, 3)$ to $(2, 1, 3)$. Their corresponding permutation matrices are

   $$x = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad y = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Prove that every permutation in $S_3$ is some product of $x$’s and $y$’s (i.e. $x$ and $y$ generate $S_3$). (Hint: note that $x^3 = y^2 = I$, the identity permutation.)

4) How hard was this homework for you? How long did it take?