
2) We have seen that a continuous map from \([0, 1]\) to \([0, 1]\) has a fixed point. Find an example of a continuous map from \((0, 1)\) to \((0, 1)\) that does not have a fixed point.

3) Let \(f\) be a real-valued function on \(\mathbb{R}\). Suppose that for a given \(x \in \mathbb{R}\),

\[
\lim_{n \to \infty} (f(x + a_n) - f(x - a_n)) = 0
\]

for all sequences \(\{a_n\}\) that converge to 0. Is \(f\) continuous at \(x\)?

4) a) Let \(S\) be a subset of \(\mathbb{R}\), and let \(f : S \to \mathbb{R}\) and \(g : \mathbb{R} \to \mathbb{R}\) be uniformly continuous functions. Prove that the composition \(g \circ f : S \to \mathbb{R}\) is uniformly continuous.

b) Let \(f\) and \(g\) be two uniformly continuous functions from \(S\) to \(\mathbb{R}\). Prove that \(f + g\) is uniformly continuous.

c) Show that there exist uniformly continuous functions \(f\) and \(g\) from \(S\) to \(\mathbb{R}\) such that the multiplication \(f \cdot g\) is not uniformly continuous.

5) How difficult was this homework? How long did it take?