1) Do problems 15.1, 15.6(a,b), 13.3, 13.5, 13.7 from the book.

2) a) By using the integral test, or otherwise, prove that

\[
\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}
\]

converges if and only if \( p > 1 \).

b) Suppose \( \{a_n\} \) is a non-increasing sequence of positive real numbers, and that \( \sum a_n \) converges. By considering the Cauchy criterion, or otherwise, prove that \( na_n \to 0 \) as \( n \to \infty \). By considering part (a), show that the converse result is not true.

3) Consider the following functions defined for \( x, y \in \mathbb{R} \):

\[
d_1(x, y) = (x - y)^2, \quad d_2(x, y) = \sqrt{|x - y|}, \quad d_3(x, y) = |x^2 - y^2|, \quad d_4(x, y) = |x - 2y|.
\]

For each function, determine whether it is a metric or not.

4) a) Two metrics \( d_1 \) and \( d_2 \) defined for \( x, y \in S \) are called topologically equivalent if a subset of \( S \) is open with respect to \( d_1 \) if and only if it is open with respect to \( d_2 \). Let \( N_r(x, d_i) \) denote the \( r \)-neighborhood of \( x \) with respect to the metric \( d_i \). Show that two metrics \( d_1 \) and \( d_2 \) on \( S \) are topologically equivalent if and only if for any \( r > 0 \) and any \( x \in S \) there exist an \( r' \) and \( r'' \) in \( \mathbb{R} \) such that

\[
N_{r'}(x, d_1) \subset N_r(x, d_2) \quad \text{and} \quad N_{r''}(x, d_2) \subset N_r(x, d_1).
\]

b) Show that if \( d_1 \) and \( d_2 \) are equivalent metrics on \( S \) and \( \{s_n\} \) is a sequence in \( S \) which converges with respect to \( d_1 \), then it converges with respect to \( d_2 \) as well.

5) How difficult was this homework? How long did it take?