Math 286 G1 — Midterm 1

This is a mock-up exam for the real one. The actual exam will provide enough space for your writing. Here, we are saving trees in Brazil.

1. Determine the orders of the following differential equations. Also find if they are linear or nonlinear. In the case of linear, decide it is the homogeneous or non-homogeneous case (Say NA if it is nonlinear).
   
   (a) \( y' - 2y^2 = 0 \)
   
   (b) \( xy'' + x^2y' - \sin(x)y = e^x \)
   
   (c) \( \cos(x)y' - \sin(x)y = 0 \)
   
   (d) \( y^{(4)} - 3y'' = \frac{5y'}{1+x^2} \)
   
   (e) \( \left( \frac{d}{dx} \right)^2 y + \left( \frac{dy}{dx} \right)^2 = \frac{d}{dx} y \)

2. State the existence and uniqueness theorem for first order differential equations with initial data
   
   \( y' = f(x, y) \quad y(a) = b \)

3. For the first order linear equation
   
   \( y' + P(x)y = Q(x) \)

   (a) Write down the formula for the integral factor, which is \( H \) in my lecture note:
   
   (b) Using the integral factor of above, solve the following equation

   \( y' + \frac{1}{x}y = 2 \)

4. For the second order linear equation
   
   \( y'' + 7y' + 10y = 0 \)

   (a) Find two solutions of the differential equation above and use the Wronskian method to show that they are linearly independent.
   
   (b) Find the particular solution which satisfies the initial data

   \( y(0) = 2, y'(0) = -1 \)

5. Using the substitution method to solve the Bernulli type equation (Section 1.6: 22)

   \( x^2y' + 2xy = 5y^4 \)
6. A motorboat weighs 32,000 lb and its motor provides a thrust of 5000 lb. Assume that the water resistance is 100 pound for each foot per second of the speed \( v \) of the boat. Then
\[
1000 \frac{dv}{dt} = 5000 - 100v
\]

(a) If the boat starts from rest, what is the maximum velocity that it can attain?

(b) Find the equilibrium solution of the differential equation above. And determine if it is a stable or non-stable solution. (Hint, consider the derivative around the equilibrium solution)

7. For the 2nd order non-homogeneous differential equation
\[
y'' - 5y' + 4y = 3x
\]

Find the general solution, both the homogeneous part and the particular part.