MATH 442 — HOMEWORK 5

Section 2.4: 19(a). Then guess a solution formula for the diffusion equation $u_t = k\Delta u$ in two dimensions, under the initial condition $u(x,y,0) = \phi(x,y)$. Show your formula is valid (meaning it satisfies the diffusion equation and the initial condition) in the special case where ϕ has separated form $\phi(x,y) = f(x)g(y)$.

Additional Problem A (Diffusion equation with lower order terms.) Suppose

$$u_t - ku_{xx} + au_x + bu = 0,$$

where $k > 0, a, b \in \mathbb{R}$ are constants. Take initial condition $u(x,0) = \phi(x)$.

Let $v(x,t) = e^{bt}u(x+at,t)$ and find the PDE satisfied by v. Solve for v, then solve for u.

Remarks.

1. This problem essentially combines Exercises 2.4.16 and 2.4.18.
2. The "convection" term au_x in the diffusion equation indicates that the diffusion takes place in a moving medium (for example in water flowing down a straight river). Where does this moving coordinate frame show up in your solution formula for u?

Additional Problem B [problem 8-23 in Mathews & Walker "Mathematical Methods of Physics"]: In an absorbing medium in a nuclear reactor, the neutron density $n(x,y,z,t)$ obeys the differential equation

$$n_t = \kappa\Delta n - \frac{n}{T}$$

where T is a constant. At $t = 0$, a burst of neutrons is produced in the yz-plane of an infinite medium, so that

$$n(x,y,z,0) = \delta(x).$$

Solve for the neutron density.

Section 3.1: 2, 3

Section 3.2: 2. Note when Strauss says "initially at rest" here, he means "initial displacement zero". (Warning. I don't understand Strauss's answer at the back of the book. Instead, I recommend using the hammer blow problem to help you solve this problem, then checking your answer with Iode.)

Section 3.2: 5 (Do this just with *pictures*, using the graphical reflection principles discussed in class. You can check your answer using Iode.)