last time: Distance in $\mathbb{R}^2$ and $\mathbb{R}^3$

$P = (x_1,y_1)$ distance between $P$ and $Q$
$Q = (x_2,y_2)$

in $\mathbb{R}^3$ $P = (x_1,y_1,z_1)$
$Q = (x_2,y_2,z_2)$
\[
\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}
\]

If $\mathbb{R}^n$ is $\mathbb{R}(x_1,...,x_n)$ $x_1,...,x_n \in \mathbb{R}^3$

$P = (x_1,...,x_n)$
$Q = (y_1,...,y_n)$
\[
\sqrt{(x_1-y_1)^2 + ... + (x_n-y_n)^2}
\]

12.2: Vectors

What is a vector? A line segment with direction. (an arrow)
line segment, not a vector.

"terminal point": this is a vector.
"initial point"
When are two vectors the same?

\[ (0,0,10) \text{ (units are feet)} \]

\[ \text{this is the same vector.} \]

\[ \text{not the same vector.} \]

2 properties of a vector:

- Length (magnitude)
- Direction

Another way to think of vectors: The difference between two points in space.

\[ \vec{v}, \text{ or } \vec{w}, \vec{AB} \]

\[ \vec{AB} = (3,2) = \langle 3,2 \rangle \]

\[ = \overrightarrow{B-A} \]

\[ = (2,5) - (-1,3) \]

\[ = (2-(-1)\delta, 5-3) \]

\[ \vec{w} = (3,2) - (0,0) = (3-0, 2-0) = (3,2) \]
Vector Addition

\( \vec{u}, \vec{v} \) vectors, then their sum, \( \vec{u} + \vec{v} \) is a vector, and \( \vec{u} + \vec{v} = \vec{v} + \vec{u} \).

parallelogram law.

\[ \vec{u} = (3, 2) \]
\[ \vec{v} = (-1, 1) \]
\[ \vec{u} + \vec{v} = (2, 3) \]

To add vectors, add in each component.

Scalar multiplication: multiplying a number with vector.

If \( \vec{v} = (v_1, v_2) \), \( c \) is a scalar
\[ c\vec{v} = (cv_1, cv_2) \].

eg \( \vec{v} = (1, 1) \), \( c = 3 \), \( 3\vec{v} = (3, 3) \).
\[ d = -1 \]
\[ d\vec{v} = (-1, -1) \]

Changes the length (if \( c \neq 1 \)) and the direction (if \( c < 0 \)).
Length of \((v_1, v_2)\) = \(\sqrt{(v_1)^2 + (v_2)^2} = |\vec{v}|\)

or \(|\vec{v}|\)

\[
\text{Length } (v_1, v_2, v_3) = \sqrt{v_1^2 + v_2^2 + v_3^2} = |\vec{v}|.
\]

Note: \(1c \vec{v} = |c| |\vec{v}|\).

Vectors of length 1 are called **unit vectors**.

If we choose \(c = \frac{1}{|\vec{v}|}\), then \(|\frac{\vec{v}}{|\vec{v}|}| = \frac{1}{|\vec{v}|} |\vec{v}| = 1\).

* We can scale any (non-zero) vector to become a unit vector.

eg \(\vec{v} = (3, 4)\). \(|\vec{v}| = \sqrt{9 + 16} = \sqrt{25} = 5\).

\[
\frac{\vec{v}}{|\vec{v}|} = \frac{1}{5} (3, 4) = \left(\frac{3}{5}, \frac{4}{5}\right). \text{ is a unit vector.}
\]

\[
\sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1.
\]
Standard basis vectors.

\[(6, -3) = 6(1, 0) + (-3)(0, 1)\]

What are vectors useful for?

Next time:
12.3 Dot product
12.5 Application of dot product.