Wednesday December 5th

Recall Thm: If $\vec{F}(x,y,z)$ is defined on all of $\mathbb{R}^3$ and $\text{curl}(\vec{F}) = \vec{0}$, then $\vec{F}$ is conservative.

Can use Stokes' Thm to prove this.

$\vec{F}$ is conservative $\iff \int_C \vec{F} \cdot d\vec{r}$ for every closed path $C$.

Proof: Assuming $\text{curl}(\vec{F}) = \vec{0}$, show $\int_C \vec{F} \cdot d\vec{r} = 0$, for any $C$.

Let $C$ be a simple closed path. Find a surface $S$ which has $C$ as its boundary curve. (For some curves it is not obvious, but it can be done). Then $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}(\vec{F}) \cdot \vec{n} \, dS = 0$.

Note: This is why we need domains to be simply connected, so that we may find disks or other surfaces bounded by curves. In fact can use this to define a simply connected region of $\mathbb{R}^3$: $E$ is simply connected if every simple closed curve $C \subseteq E$ is the boundary of a simple surface $D$ inside $E$.

eg simply-connected: ball $\cup$ cube $\sqcup$ "fat sphere" $\cup$ $x^2 + y^2 + z^2 \leq 2$.

not s-c: solid torus $\cup$ as core curve doesn't bound a surface.
Q: Why does every simple closed curve \( C \subset \mathbb{R}^3 \) bound an orientable surface?

If \( C \) is not "knotted" then it bounds a disk, so problem is knotted curves. Leads to studying knot theory.

eg. \( \text{trefoil knot} \)

first guess: \( \text{but this surface is not orientable} \)