1. Use the first phase of the two-phase simplex method to show that the following linear program is infeasible.

Minimize \( z = 3x_1 + x_2 + 2x_3 \)

subject to

\[
\begin{align*}
  x_1 + 3x_2 + 5x_3 - x_4 &= 10, \\
  2x_1 - x_2 - 9x_3 - x_4 &= 1, \\
  4x_1 + 5x_2 + x_3 + x_4 &= 7, \\
  x_1, \ldots, x_4 &\geq 0.
\end{align*}
\]

2. Suppose that you are solving an LP in standard form with 5 variables \( x_1, \ldots, x_5 \) and 2 constraints and the following objective function

\[ \min z = x_1 + 7x_2 + 5x_3 + x_4 + 6x_5. \]

You add two artificial variables \( y_1 \) and \( y_2 \) and after the first phase of the two-phase simplex method you have the following tableau

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\xi)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>13</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-8</td>
<td>-13</td>
<td>-14</td>
<td>-3</td>
</tr>
</tbody>
</table>

Drive the artificial variable out of the basis and then solve the original linear program by doing phase two of the simplex method.

3. You can use a calculator or computer algebra system to help with this problem, but you must show all your steps.

Suppose that at a stage of the simplex algorithm for a minimization problem, we have the following tableau \( \bar{T} \):

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-z)</td>
<td>8</td>
<td>0</td>
<td>8/3</td>
<td>-11</td>
<td>0</td>
<td>4/3</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>4</td>
<td>1</td>
<td>2/3</td>
<td>0</td>
<td>0</td>
<td>4/3</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>2</td>
<td>0</td>
<td>-7/3</td>
<td>3</td>
<td>1</td>
<td>-2/3</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>2</td>
<td>0</td>
<td>-2/3</td>
<td>-2</td>
<td>0</td>
<td>2/3</td>
</tr>
</tbody>
</table>
You are told that the inverse of the current basis is

\[ A_B^{-1} = [A_1, A_4, A_6]^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 2 \\ -1 & 2 & 1 \end{pmatrix} \]

and

\[ c_B^T = [c_1, c_4, c_6] = [-1, -3, 1]. \]

Find vectors \( c \) and \( b \) and the matrix \( A \) that correspond to the original linear program.

4. Questions on Big O notation and LU factorization
   (a) True or False: \( g(n) = n^3 \) is \( O(n^2) \). Explain using the definition of \( O(f(n)) \).
   (b) True or False: \( h(n) = n^{1/100} \) is \( O((\log n)^{100}) \).
   (c) Compute a LU factorization of the matrix \( M \) shown below. Use forwards and backwards substitution with your LU factorization to solve \( Mx = [1, 10, 100]^T \). (For this simple example, it might be easier to solve the problem directly, but please use the LU factorization method.)

\[
M = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ -1 & 0 & 1 \end{pmatrix}.
\]