1. A company produces three types of chemicals: chemical A, chemical B and chemical C. They sell chemical A for $30 per barrel, chemical B for $20 per barrel, and chemical C for $10 per barrel. Chemical A requires .2 units of energy and 5 units of raw material to produce, Chemical B requires .3 units of energy and 6 units of raw material to produce and Chemical C requires .2 units of energy and 3 units of raw material to produce. Assume that all of the chemicals the company produces are sold. The company must produce at least 25 barrels of chemicals per day. It can use at most 7 units of energy per day and at most 120 units of raw material each day. The company wishes to maximize its profits. Formulate the appropriate linear program in standard form.

2. Solve the following problem, i.e. if the following program has a optimal solution you must find an optimal solution and compute the value of the objective function at the optimal solution, otherwise you must state whether the program is infeasible or unbounded. You should draw the feasible region in the plane.

$$z = x_1 + 2x_2 \rightarrow \text{max}$$

with respect to

$$\begin{cases} 
   x_1 + 4x_2 & \leq 12, \\
   x_1 + x_2 & \leq 4, \\
   5x_1 + 2x_2 & \leq 15, \\
   x_1, x_2 & \geq 0. 
\end{cases}$$

3. State (but do not solve) the following LP in standard form. Note that the variable $x_3$ is not constrained to be nonnegative.

$$z = -x_1 + 2x_2 - 3x_3 \rightarrow \text{max}$$

with respect to

$$\begin{cases} 
   4x_1 + 2x_2 + 2x_3 & \leq 3, \\
   x_1 + x_2 + 4x_3 & \geq -7, \\
   2x_1 - 3x_2 & \leq 5, \\
   x_1, x_2, & \geq 0. 
\end{cases}$$

4. Suppose that there exists $x_0 \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$ such that $x_0$ is feasible for the following linear program $P$

$$\begin{align*}
   \min & \quad c^T x \\
   \text{subject to} & \quad Ax = b \\
   & \quad x \geq 0
\end{align*}$$
and \( y \) satisfies

\[
\begin{align*}
c^T y &< 0 \\
Ay & = 0 \\
y & \geq 0.
\end{align*}
\]

Prove that \( P \) is unbounded.

5. (a) In \( \mathbb{R}^2 \), let \( C \) be the square \( \{ (x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1 \} \). Show that \( (1, 1) \) is a vertex of \( C \).

(b) Prove that if a point \( v \) is a vertex of a convex polyhedron \( P \), then it must be an extreme point of \( P \). (The converse is also true, but you do not need to prove it for this homework.)