Please take a moment to just breathe.

In this lecture, we introduce the derivative and use it to compute how fast a pumpkin thrown off the Altgeld bell tower will be traveling when it hits the ground.

Note to students: Do NOT throw pumpkins (or anything else) off the Altgeld bell tower.

Suppose you drive up to Chicago. The graph below shows you miles north of Altgeld at time $t$. 

- You hit traffic in Chicago.
- You forgot something and went back home.
Suppose your position at time \( t \) is given by the function \( p(t) \).

Then the average velocity on the time interval \((a, a + \Delta t)\) is

\[
\frac{p(a + \Delta t) - p(a)}{\Delta t}.
\]

**Example: Average velocity**

Suppose that on the interstate, driving through the cornfields, you drive at the local speed limit.

If you measure your position after 1 hour and after 3 hours, you get

\[
p(1) = 10
\]
\[
p(3) = 140
\]

Your average speed on the interstate is

\[
\frac{p(3) - p(0)}{3 - 1} = \frac{130}{2} \text{ mph}, \text{ or } 65 \text{ mph} \quad (\text{about } 105 \text{ kph})
\]

However, suppose that when you hit Chicago traffic, you slow down:

\[
p(3) = 140
\]
\[
p(4) = 160
\]

Your average speed then would be

\[
\frac{p(4) - p(3)}{4 - 3} = \frac{20}{1} = 20 \text{ mph} \quad (\text{about } 32 \text{ kph})
\]

How fast are you going right at \( t = 3 \)?

What does your speedometer say at 3pm?

You could compute your average speed in the one second between 3:00:00 and 3:00:01.

Or you could compute it over one millisecond, or one microsecond, or one nanosecond, or one picosecond....
The **instantaneous velocity** of an object with position \( p(t) \), at a specific time \( t = a \), is

\[
\lim_{\Delta t \to 0} \frac{p(a + \Delta t) - p(a)}{\Delta t}
\]

Note that this quantity also represents the slope of the tangent line to the graph of \( y = p(x) \) of at \((a, p(a))\).

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**Example:** Suppose we drop a pumpkin off of the Altgeld bell tower, and suppose its position at time \( t \) is given by

\[
p(t) = 40 - 4.9t^2
\]

where \( t \) is in seconds and \( p(t) \) is in meters.

What is the pumpkin's velocity at time \( t = 1 \)?

If \( p(t) = 40 - 4.9t^2 \), find \( p'(1) \). We'll use "\( h \)" for "\( \Delta t \)."

\[
p'(1) = \lim_{h \to 0} \frac{p(1 + h) - p(1)}{h}
\]

\[
= \lim_{h \to 0} \frac{40 - 4.9(1 + h)^2 - (40 - 4.9(1)^2)}{h}
\]

\[
= \lim_{h \to 0} \frac{-4.9(2 + h) + 4.9}{h}
\]

\[
= \lim_{h \to 0} \frac{-4.9(2 + h) + 4.9}{h} = \lim_{h \to 0} -4.9(2 + h) = -9.8 \text{ m/s}
\]
To find $p'(a)$ for a more general time $t = a$:

$$p'(a) = \lim_{h \to 0} \frac{p(a + h) - p(a)}{h}$$

$$= \lim_{h \to 0} \frac{40 - 4.9(a + h)^2 - 40 + 4.9(a)^2}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(a^2 + 2ah + h^2) + 4.9a^2}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(2ah + h^2)}{h} = \lim_{h \to 0} -4.9(2a + h) = -9.8a$$

How fast is the pumpkin moving when it hits the ground?

$p(t) = 40 - 4.9t^2$

It hits the ground when $40 - 4.9t^2 = 0$, which happens at $t = \sqrt{\frac{40}{4.9}}$.

The velocity at that time is

$$p'(\sqrt{\frac{40}{4.9}}) = -9.8 \sqrt{\frac{40}{4.9}} = -28 \text{ m/s}$$

or about 101 kph or 63 mph downwards.