MORE VOLUME INTEGRALS

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) The shaded region is bounded by the curves $y_1 = x^4 - 2x$ and $y_2 = 3x^3 - 3x^2$ over $[0, 2]$.

(a) Give an integral (do not evaluate) for the area of the region.

\[ \text{Area} = \int_0^2 (3x^3 - 3x^2) - (x^4 - 2x) \, dx \]

(b) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $y = -3$.

Vertical Slices

Washers

\[ A = \pi \left( r_{out}^2 - r_{in}^2 \right) \]

Vertical Slices

Cylindrical Shell

\[ V_{ol} = 2\pi \int_0^2 (8-x) \left( (3x^3 - 3x^2) - (x^4 - 2x) \right) \, dx \]

(c) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $x = 8$.

\[ V_{ol} = \pi \int_{y_1}^{y_2} ((8-x)^2) \left( (3x^3 - 3x^2) - (x^4 - 2x) \right) \, dy \]

\[ y_1(x) = 1x^4 - 2x; \quad y_2(x) = 3x^3 - 3x^2 \]

The cubic is on top at $x = 1$.
(d) Use cylindrical shells to find the volume of a right circular cone with height $h$ and with base radius $r$. You should get $\frac{1}{3} \pi r^2 h$.

$$\text{Vol} = 2\pi \int_0^r (x) (h - \frac{h}{r} x) \, dx = 2\pi \int_0^h x - \frac{h}{r} x^2 \, dx$$

$$= 2\pi \left[ -\frac{h}{2} x^2 - \frac{h}{3r} x^3 \right]_0^r = 2\pi \left( \frac{h}{2} r^2 - \frac{h}{3r} r^3 \right) - 2\pi (0)$$

$$= h\pi r^2 - \frac{2}{3} h\pi r^2 = \frac{1}{3} \pi r^2 h$$

3) An 800 pound gorilla going up 20 ft takes 1600 ft-lbs of work.

4) An .5 kg slow loris going up 2 meters takes $9.8(.5)(2) = 9.8$ Newton-meters (or Joules) of work.