**Volume Integrals**

1. For the following problems, set up integrals which calculate the volume of the described solid. **DO NOT EVALUATE** these integrals.

   a. The solid obtained by rotating the region bounded by \( y = x^2 - 4x + 5, \ x = 1, \ x = 4, \) and \( y = 0 \) about the \( x \)-axis.

   \[
   r = x^2 - 4x + 5 \\
   A = \pi r^2 \\
   \int_1^4 \pi (x^2 - 4x + 5)^2 \, dx
   \]

   b. Determine the volume of the solid obtained by rotating the region bounded by \( y = x^2 - 2x \) and \( y = x \) about the line \( y = 4 \).

   \[
   r = 4 - x \\
   R = 1 - (x^2 - 2x) \\
   A = \pi R^2 - \pi r^2 \\
   \int_0^3 \pi \left( 4 - x^2 + 2x \right)^2 - \pi (4 - x)^2 \, dx
   \]

   c. The solid obtained by rotating the region bounded by \( y = \sqrt{x} \) and \( y = \frac{x}{4} \) that lies in the first quadrant about the \( y \)-axis.

   \[
   r = \sqrt{x} \Rightarrow x = y^3 \\
   A = \pi R^2 - \pi r^2 \\
   \int_0^2 \pi (4y)^2 - \pi (y^3)^2 \, dy
   \]
(2) Let \( R \) be the region bounded by the graph of \( y = e^x \) and the \( x = 0 \) and \( x = 3 \). Using the disk/washer method, set up a “dy” integral for the volume of the solid formed by revolving \( R \) around the \( y \)-axis. (Hint: You will need 2 integrals).

\[
\int_1^3 \pi (3)^2 \, dy + \int_1^2 \pi (3)^2 - \pi (\ln y)^2 \, dy
\]

(3) Write a definite integral that represents the following volumes.

(a) Slices perpendicular to the \( x \)-axis are squares over the area bounded by \( 2x - x^2 \) and the \( x \)-axis.

(b) Slices perpendicular to the \( x \)-axis are equilateral triangles over the area bounded by \( y = x \) and \( y = \sqrt{x} \).

(c) Slices perpendicular to the \( y \)-axis are equilateral triangles over the area bounded by \( y = x \) and \( y = \sqrt{x} \).

(4) (Not for turning in: if there is time) Compute the volume of a “rugby ball” if it has length 300 mm and circumference 600 mm and it is formed by rotating an ellipse about its major axis (these are actual official dimensions).