AREAS BETWEEN CURVES

Solutions

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) Write an integral (BUT DO NOT EVALUATE) for the area bounded by \( y = x + 5 \) and \( y = x^2 - 1 \). Sketch a graph of these region as well.

\[
\text{Intersection} \\
\begin{array}{c}
  x + 5 = x^2 - 1 \\
  x^2 - x - 6 = 0 \\
  (x - 3)(x + 2) = 0 \\
  x = 3, -2
\end{array}
\]

\[
\text{Area} = \int_{-2}^{3} (x + 5) - (x^2 - 1) \, dx
\]

(2) The region bounded by \( y = \sqrt{x} \) and \( y = x^2 \) can be expressed as an integral with respect to both \( y \) and \( x \). What are these integrals?

\[
\text{Intersection} \\
\begin{array}{c}
  y = \sqrt{x} \\
  y = x^2
\end{array}
\]

\[
y = \sqrt{x} \\
x = y^2
\]

\[
\text{Area} = \int_{0}^{1} (\sqrt{x}) - (x^2) \, dx
\]

\[
= \int_{0}^{1} (y) - (\sqrt{y}) \, dy
\]

(3) Write an integral (BUT DO NOT EVALUATE) for the area bounded by \( y = \sin x \) and \( y = \cos x \) from \( \pi/4 \) to \( 5\pi/4 \).

\[
\text{Area} = \int_{\pi/4}^{5\pi/4} (\sin(x)) - (\cos(x)) \, dx
\]
(4) Let $R$ be the finite region bounded by $x = y^2$, $x = (y - 6)^2$, and $x = 0$. Set up two definite integrals which represent the area of the region $R$ - with one integral integrating with respect to $y$, and the other with respect to $x$.

\[
\text{Area} = \int_0^3 (-\sqrt{x} + 6) - (\sqrt{x}) \, dx
\]

\[
= \int_0^3 y^2 \, dy + \int_3^6 (y-6)^2 \, dy
\]

(5) Let $R$ be the finite region bounded by $y = 2 \ln x$, $y = 3 \ln x$ and $y = 6$. Determine the area of $R$ by evaluating a definite integral with respect to $y$.

\[
\text{Area} = \int_0^6 (e^{\frac{12}{y}}) - (e^{\frac{18}{y}}) \, dy
\]

(6) Consider the finite region $R$ bounded by $y = 3e^{2x} + 1$, $y = 17 - e^{2x}$ and the $y$-axis. Determine the area of $R$ by evaluating a definite integral with respect to $x$. You should simplify your final answer.

\[
\text{Intersection: } 3e^{2x} + 1 = 17 - e^{2x} \quad \Rightarrow \quad x = \frac{\ln(4)}{2}
\]

\[
\text{Area} = \int_0^{\frac{\ln(4)}{2}} (17 - e^{2x}) - (3e^{2x} + 1) \, dx
\]

(7) Set up, but do not evaluate, one or more definite integrals with respect to $y$ to represent the area of $R$ in the previous problem.

\[
\text{Area} = \int_0^{\frac{\ln(\frac{17}{2})}{2}} \left( \frac{\ln(\frac{17-y}{2})}{2} \right) \, dy + \int_{\frac{\ln(3)}{2}}^{\frac{\ln(\frac{17}{2})}{2}} \left( \frac{\ln(17-y)}{2} \right) \, dy
\]