Optimization Word Problems

1. What is the area of the largest rectangle that can fit with one edge on the $x$-axis, one edge on the $y$-axis, the lower left corner at the origin, and the upper right corner on the curve $y = 2 - x$?

2. Which point on the graph of $y = \sqrt{5x}$ is closest to the point $(10, 0)$?
3. A company wants to manufacture cylindrical aluminum cans with a volume of 1000cm³ (1 liter). What should the radius and height of the can be to minimize the amount of aluminum used?

**Volume of a Cylinder:** \( V = \pi r^2 h \)

**Surface Area of a Cylinder:** \( S = 2\pi r^2 + 2\pi rh \)

4. A poster contains 1000cm² of printed area in the middle, with 4cm margins each at top and bottom, and 2cm margins at each side. What overall dimensions would minimize the total area of the poster?
5. Show that the point between two posts of fixed lengths $A$ and $B$ which minimizes the distance $\alpha + \beta$ has the property that $\frac{a}{b} = \frac{A}{B}$. 