Part 1

1) A critical point of \( f(x) \) is a number \( c \) in the domain of \( f(x) \) such that \( f'(c) = 0 \) or DNE.

2) \( f(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3) \)
\[ f'(x) = 0 \implies x = 1 \text{ or } 3 \]

\( f(-1) = -16 \)
\( f(1) = 4 \)
\( f(3) = 0 \)
\( f(5) = 25 \)

\( \implies f(x) \) has an abs. max at \( x = 5 \)
and an abs. min at \( x = -1 \)
\[ f'(x) = 3 (x + 4)^{2/3} + 3x \cdot \frac{2}{3} (x + 4)^{-1/3} \]
\[ = (x + 4)^{-1/3} (3(x + 4) + 2x) = \frac{1}{\sqrt[3]{x+4}} (5x+12) \]

critical points: \( x = -4, x = -\frac{12}{5} \)

\[ f(-4) = 0 \]
\[ f(-\frac{12}{5}) = -3 \]
\[ f(-1) = -3 \cdot \frac{23}{5} \]

\( \Rightarrow f(x) \) has an abs. max
\[ a+ \text{ at } x = -4 \]

and an abs. min
\[ a+ \text{ at } x = -\frac{12}{5}. \]

Part 2

1) \[ f'(x) = (x - 1)(x - 2)^2 (x - 3) \]

\[ f' \quad + \quad - \quad - \quad + \quad \to x \]

\[ f \quad (\nearrow \quad \searrow \quad \searrow \quad \nearrow \quad \nearrow) \]

thus \( f(x) \) is increasing on \((-\infty, 1) \) and \((3, \infty)\)

and decreasing on \((1, 3)\)

local maxima at \( x = 1 \)

local minima at \( x = 3. \)
Thus $f(x)$ is increasing on $(-\infty, 0), (2.1, 3.9)$

and decreasing on $(0, 2.1)$ and $(3.9, 5.5)$

Local maxima at $x = 0, 3.9$

Local minima at $x = 2.1$

$$f(x) = \frac{\ln(x)}{3x} \quad \text{domain } x > 0$$

$$f'(x) = \frac{1 - \ln(x)}{3x^2}$$

Thus $f(x)$ is increasing on $(0, e)$

and decreasing on $(e, \infty)$

Local maxima at $x = e$

does not have local minima
\[ f(x) = (x^2 + 1)e^{-x} \] is a differentiable function and it increases most rapidly (\( \Rightarrow \) absolute max of \( f'(x) \)).

\[ f'(x) = 2xe^{-x} - e^{-x}(x^2 + 1) = -e^{-x}(x^2 - 2x + 1) \]
\[ = -e^{-x}(x-1)^2 \]

So we want to find the absolute max of \( g(x) = -e^{-x}(x-1)^2 \)

\[ g'(x) = e^{-x}(x^2 - 4x + 3) = e^{-x}(x-1)(x-3) \]

\[ g' \quad + \quad - \quad + \quad + \]

so at \( x = 1 \) \( g(x) \) has a local maxima, but is it absolute maxima? we know that \( g(x) \) looks like this

so we need to find out if this part outgrows \( g(x) \)

at \( x = 1 \) \( g(1) = 0 \) and \( \lim_{x \to \infty} g(x) = 0 \)

so \( g(x) \) has absolute maxima at \( x = 1 \) and looks roughly like...
$f'(x)$ (or $g(x)$)

$f(x)$