Derivatives as functions, and derivatives of $x^r$ and $e^x$

Instructions. Please work on these with your group and then upload your own completed worksheet.

1. Which graph might best represent the following physical scenarios?

   (a) The position of a car at a stop sign.
   (b) The position of a car coming to a stop.
   (c) The velocity of a car coming to a stop.
   (d) The position of a car leaving a stop sign.
   (e) The velocity of a car leaving a stop sign.
   (f) The position of a car traveling on the expressway.
   (g) The velocity of a car traveling on the expressway.

2. Match the 3 functions below to their derivatives. Do this by considering when a function is increasing its derivative is positive, when a function is decreasing its derivative is negative, and when a function has a min/max its derivative is 0.
3. What derivative does \( \lim_{h \to 0} \frac{2h - 1}{h} \) represent? Sketch the function on the interval \([-2,2]\), and draw the tangent line at the relevant point.

4. Using the derivative rules (and not the limit definition of derivative), find the derivatives of the following functions.
   Hint: you do not need the chain rule!
   (a) \( q(x) = \left( \frac{x \sqrt{x}}{\sqrt{x}} \right)^{12} + \ln \left( \frac{x^3}{e^2 + 4} \right) = \left( x \right)^{12} + \ln \left( \frac{x^3}{e^2 + 4} \right) = x^{12} + \ln \left( \frac{x^3}{e^2 + 4} \right) \)
   
   (b) \( f(x) = 8x^{-1/3} + \frac{6}{x^{-4/3}} = 8x^{-1/3} + 6x^{4/3} \)

5. Find the values of \( m \) and \( b \) that make this function differentiable everywhere.

6. Determine the equation of the line which is tangent to \( f(x) = 7 - 5e^x \) and perpendicular to \( y = \frac{1}{10}x + 3 \).

7. **Very Useful Fact:**

   If \( f \) is differentiable at \( x = a \), then \( f(a + h) = f(a) + f'(a)h + E(h)h \) for some function \( E(h) \) with limit 0 at 0.

   For instance, for \( f(x) = \sqrt{x} \) at \( x = 1 \), we get \( \sqrt{1+h} = 1 + \frac{h}{2} + E(h)h \).

   This gives us a good approximation for small values of \( h \): \( \sqrt{1+h} \approx 1 + \frac{h}{2} \).

   Show that the Very Useful Fact follows directly from the definition of the derivative.