Solutions to worksheet 3

1. Show $E(h) = 4h$ has limit 0 at 0.

We need to show:

given $\epsilon > 0$, there exists $\delta > 0$ so that
if $0 < |h| < \delta$, then $|E(h)| < \epsilon$.

Suppose $\epsilon = 0.8$

If we want $|E(h)| < 0.8$, or $-0.8 \leq 4h \leq 0.8$,
then we need to take: $-0.2 \leq h \leq 0.2$

So $\delta$ can be $0.2$ (or smaller).

General strategy: let $\delta = \epsilon/4$

Then given $\epsilon$, let $\delta = \epsilon/4$.
If $0 < |h| < \delta$, then $|E(h)| = 14h| < 4\delta = \epsilon.$
2. Show \( E(h) = h^2 \) has limit 0 at 0.

Suppose \( \varepsilon = \frac{1}{100} \).

If we want \( |E(h)| < \frac{1}{100} \),

we want

\[-\frac{1}{100} < h^2 < \frac{1}{100},\]

red herring (irrelevant)

Since

\( h^2 > 0 \)

for all \( h \)

we need

\[ |h| < \frac{1}{10}. \]

General strategy: let \( \delta = \sqrt{\varepsilon} \).

Then given \( \varepsilon \), let \( \delta = \sqrt{\varepsilon} \).

Then if \( 0 < |h| < \delta \), then \( |E(h)| = |h^2| < \delta^2 = \varepsilon \).

3. We have a frequency function \( F(p) \) that depends on pressure. We need frequencies within \( \varepsilon \) of 60.

Any pressure in the range \((20-a, 20+b)\) shown in the diagram will suffice.

Pick \( \delta = \min(a, b) \).

Then \((20-\delta, 20+\delta)\) is a subset of \((20-a, 20+b)\), so any pressure in this range will suffice.
Find largest \( \delta \) that will work for \( \epsilon = 0.6 \) in \( \lim_{x \to 2.5} f(x) = 3 \).

As before, we see that to get \( f(x) \) in \((2.4, 3.6)\), we need to choose \( x \) in \((1.6, 3.1)\).

To find \( \delta \), find an interval that is symmetric about 2.5 that is contained in \((1.6, 3.1)\):

Symmetric interval: \((-1.9, 3.1)\)

So \( \delta \) could be 0.6.

Note: any smaller \( \delta \) will also work:

\((2.499, 2.501)\) is also contained in \((1.6, 3.1)\).

\(\delta\)

To check that \( \lim_{x \to a} f(x) = L \),

we need to check that \( E(\epsilon) = |f(a+h) - L| \)

has limit 0 as \( h \to 0 \).

For \( \lim_{x \to 2} (4x + 10) = 18 \)

\( E(\epsilon) = |4(2+h) + 10 - 18| \)

\( = 4\epsilon \)

We saw in problem 1 that \( 4\epsilon \) has limit 0 as \( h \to 0 \), so we're done.
(6) Show \( \lim_{x \to 3} (x^2) = 9 \).

We need to check that
\[
E(h) = f(3+h) - L \text{ has limit } 0 \text{ at } 0
\]

\[
E(h) = (3+h)^2 - 9 \\
= 9 + 6h + h^2 - 9 \\
= 6h + h^2.
\]

We saw that \( h^2 \) has limit 0 at 0. in prob(2).
We saw that 4h has limit 0 at 0 in prob(1),
and the proof is the same.

Since \( E(h) \) is the sum of two functions with limit 0 at 0, \( E(h) \) also has limit 0 at 0,
by the sum rule.