1. Find the derivative of \( G(x) = \int_0^{2x} \cos(1 + t^2) dt \).

(Solution):

Let \( F(x) := \int_0^x \cos(1 + t^2) dt \). Then by the Fundamental Theorem of Calculus, we have that

\[
F'(x) = \cos(1 + x^2)
\]

Observe that \( G(x) = F(2x) \). Thus by chain rule, we see that

\[
G'(x) = 2 \cdot F'(2x) = 2 \cdot \cos(1 + 4x^2)
\]

2. Suppose the rate of change of a quantity \( W \) is

\[
W'(t) = t \sqrt{1 + t^2}
\]

What is the total net change in \( W \) between \( t = 0 \) and \( t = 4 \)?

(Solution):

We have \( W'(t) = t \sqrt{1 + t^2} \) and we want to find \( W(4) - W(0) \). Then by the net change theorem, we see that

\[
W(4) - W(0) = \int_0^4 W'(t) dt = \int_0^4 t \sqrt{1 + t^2} dt = \left[ \frac{1}{3} (1 + t^2)^{3/2} \right]_0^4 = \frac{1}{3} \left( 17^{3/2} - 1 \right)
\]

3. Compute \( \int \frac{x^5}{4 + x^2} dx \).

(Solution):

Let \( u = x^2 + 4 \) then we see that \( du = 2xdx \). Thus we have

\[
\int \frac{x^5}{4 + x^2} dx = \int \frac{(u - 4)^2}{2u} du = \int \frac{u}{2} du - \int 4 du + \int \frac{8}{u} du
\]

\[
= \frac{u^2}{4} - 4u + 8 \ln(u) + C = \frac{(x^2 + 4)^2}{4} - 4(x^2 + 4) + 8 \ln(x^2 + 4) + C
\]

\[
= \frac{x^4}{4} - 2x^2 + 8 \ln(x^2 + 4) + D
\]

Done.

4. Compute \( \int_0^{\pi/2} \frac{\cos x}{\sin x} dx \).

(Solution):

Let \( u = \sin x \). Then \( u \) changes from 0 to 1 as \( x \) changes from 0 to \( \pi/2 \). So we have

\[
\int_0^{\pi/2} \frac{\cos x}{\sin x} dx = \int_0^1 \frac{du}{e^u} = \int_0^1 e^{-u} du
\]

\[
= [-e^{-u}]_0^1 = 1 - \frac{1}{e}
\]