1. Squeeze Theorem
Since $2 \leq f(x) \leq 5$, $x^2 \geq 0$, we have
$$2x^2 \leq x^2 f(x) \leq 5x^2$$
We also have
$$\lim_{x \to 0} 2x^2 = 0$$
$$\lim_{x \to 0} 5x^2 = 0$$
Since $x^2 f(x)$ is bounded by those two functions with corresponding limits are 0, according to Squeeze theorem,
$$\lim_{x \to 0} x^2 f(x) = 0$$

2. Limits as $x$ goes to infinity
$$\lim_{x \to \infty} \sqrt{9x^2 + x} - 3x = \lim_{x \to \infty} \frac{(\sqrt{9x^2 + x} - 3x)(\sqrt{9x^2 + x} + 3x)}{(\sqrt{9x^2 + x} + 3x)}$$
$$= \lim_{x \to \infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2 + x} + 3x}$$
$$= \lim_{x \to \infty} \frac{x}{\sqrt{9x^2 + x} + 3x}$$
$$= \lim_{x \to \infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3}$$
$$= \frac{1}{6}$$

3. Intermediate Value Theorem
$$f(1) = 1 + 1 - 10 = -8 < 0$$
$$f(2) = 16 + 2 - 10 = 8 > 0$$
Since $f(x)$ is continuous on the closed interval $[1, 2]$, and 0 is a number between $-8$ and 8, by Intermediate Value Theorem, there exists some number $c$ lying in the interval $(1, 2)$, such that $f(c) = 0$. Therefore, $f(x)$ has at least one root on the interval $[1, 2]$. 
4. epsilon-delta

$|f(x) - L| < \epsilon$

The largest value of $\delta$ should be 0.04

**Reason** The first one is just by definition.

For the second one, by definition, $\lim_{x \to 0.37} f(x) = 0.25$ means that if $\epsilon = 0.15$, there exists $\delta > 0$, such that whenever $|x - 0.37| < \delta$, we have

$$|f(x) - 0.25| < 0.15$$

$$0.1 < f(x) < 0.4$$

Form the graph, we see that in order to make $0.1 < f(x) < 0.4$, $x$ should satisfy

$$0.29 < x < 0.41$$

Since

$$0.37 - \delta < x < \delta + 0.37$$

can imply $0.29 < x < 0.41$

Then we must have $\delta + 0.37 \leq 0.41$ and $0.37 - \delta \geq 0.29$. Then $\delta \leq 0.04$

Therefore, the largest value of $\delta$ should be 0.04