Math 221
Week 11 part 4
The indefinite integral
Please take a moment to just breathe.
In this section we discuss the difference between indefinite integrals and definite integrals.

We also list some of the integrals you should know.
Recall the definition:

The **definite integral** of \( f(x) \) from \( x = a \) to \( x = b \) is

\[
\int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x
\]

if this limit exists.

We can think of the integral as the signed area under the curve \( y = f(x) \) between \( x = a \) and \( x = b \).
Recall: Fundamental Theorem of Calculus (part 2):

If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(t) \, dt = F(b) - F(a)$$

where $F$ is any antiderivative of $f$. 
The **indefinite integral** of $f(x)$ represents the general form for antiderivatives of $f$. We write

$$\int f(x) \, dx = F(x) + C$$

where $F$ is a particular antiderivative of $f$, and $C$ is an unspecified constant.

Example: $\int 2x \, dx = x^2 + C$
Comparison of the definite and indefinite integrals:

<table>
<thead>
<tr>
<th>Definite Integral</th>
<th>Indefinite integral</th>
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| \[
\int_{a}^{b} f(x) \, dx
\] | \[
\int f(x) \, dx
\] |
| \[= F(b) - F(a)\] | \[= F(x) + C\] |
| a number | a family of functions |
| meaning: signed area under graph of \(f\) from \(x = a\) to \(x = b\) | meaning: antiderivative \(F'(x) = f(x)\) |
Here are some indefinite integrals you should know:

\[
\int A \, dx = Ax + C
\]

\[
\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad \text{for } r \neq -1
\]

\[
\int \frac{1}{x} \, dx = \ln |x| + C
\]

\[
\int e^{kx} \, dx = \frac{1}{k}e^{kx} + C
\]

\[
\int a^x \, dx = \frac{a^x}{\ln a} + C \quad \text{for } a > 0
\]
\[ \int \sin x \, dx = - \cos x + C \]

\[ \int \cos x \, dx = \sin x + C \]

\[ \int \sec^2 x \, dx = \tan x + C \]

\[ \int \sec x \tan x \, dx = \sec x + C \]

\[ \int \csc^2 x \, dx = - \cot x + C \]
\[
\int \frac{1}{1 + x^2} \, dx = \arctan x + C
\]

\[
\int \frac{1}{\sqrt{1 - x^2}} \, dx = \arcsin x + C
\]

\[
\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + C, \text{ for any function } u(x)
\]

We will learn the following “anti-chain-rule” next week:

\[
\int f''(u(x))u'(x) \, dx = f(u(x)) + C
\]