Consider the following questions:

- Compute the partial derivatives of \( f \).
- Compute the distortion in the horizontal direction.
- Compute the distortion in the vertical direction.
- Compute the distortion in the direction \((1, 1)\).
- Is the distortion the same in every direction at a given point?
- Are the partial derivatives of \( f \) perpendicular at each point?
- Is \( f \) conformal?
- Is it a local isometry?
- Is \( f \) one-to-one?
- Is \( f \) onto?
- Does \( f \) take lines to lines?
- Describe in words what the map does.

(1) Answer these questions for the function \( f \) from \((-\pi, \pi) \times (-1, 1) \subset \mathbb{E}^2 \) to \( S^2 \) defined as follows (we are using \((u, v)\) as coordinates on \( \mathbb{E}^2 \) rather than the usual \((x, y)\)):

\[
F(u, v) := (\sqrt{1 - v^2} \cdot \cos u, \sqrt{1 - v^2} \cdot \sin u, v).
\]

(2) Let \( g : \mathbb{R} \to \mathbb{R} \) be any differentiable function and let \( G(x, y) = (x, g(y)) \). Answer the questions (or as many as possible) for the new function

\[
H(x, y) = F \circ G(x, y) = (\sqrt{1 - g(y)^2} \cdot \cos x, \sqrt{1 - g(y)^2} \cdot \sin x, g(y)).
\]

Use the chain rule:

\[
H_x = (F \circ G)_x = F_u u_x + F_v v_x,
\]

\[
H_y = (F \circ G)_y = F_u u_y + F_v v_y,
\]

remembering that \((u, v) = G(x, y) = (x, g(y))\).

Your answer will, of course, depend on \( g \).