Quiz 6

Name: Solutions.

① Let $W = \text{span}\left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix} \right\}$ and $\vec{y} = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 13 \end{pmatrix}$.

(a) Find a point in W closest to $\vec{y}$.

(b) Find the minimum distance of $\vec{y}$ from the subspace W.

(a) We first note that $\underbrace{\begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}}_{\vec{u}_1} \cdot \underbrace{\begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}}_{\vec{u}_2} = 0$

i.e $\left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix} \right\}$ forms an orthogonal basis of W.

We need to find the orthogonal projection $\hat{y}$ of $\vec{y}$ on W.

$$\hat{y} = \left(\frac{y \cdot u_1}{u_1 \cdot u_1} \right) \vec{u}_1 + \left(\frac{y \cdot u_2}{u_2 \cdot u_2} \right) \vec{u}_2$$

$$= \left(\frac{30}{10} \right) \vec{u}_1 + \left(\frac{26}{26} \right) \vec{u}_2$$

$$= 3\vec{u}_1 + \vec{u}_2 = \begin{pmatrix} 3 \\ -6 \\ -3 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ -3 \\ 9 \end{pmatrix}$$

(b) Min distance $= \|\vec{z}\| = \| y - \hat{y} \|$

$$= \left\| \begin{pmatrix} 4 \\ 4 \\ 4 \\ 4 \end{pmatrix} \right\| = \sqrt{64} = 8$$