Quiz 5 Name: Solutions

① Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 4 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(a) Find the eigen values of A.

(b) Is A diagonalizable ?

(c) Without computing the characteristic polynomial find all eigen values of $A + I$.

(a) Need to solve $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ 4 & 3-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \quad \Rightarrow (1-\lambda)\left((3-\lambda)^2 - 4\right) = 0$$

$$\Rightarrow \lambda = 1, \quad (3-\lambda)^2 = 4$$

$$\Rightarrow \lambda = 1, \quad \lambda = 3 \mp 2$$

$$\Rightarrow \lambda = 1, 1, 5$$

(b) We need to prove that A has 3 independent eigen vectors

For $\lambda = 5$, A ~~has~~ will have one independent eigen vector.

So we need to ~~show~~ find if $A - \lambda I$ has 2 free variables for $\lambda = 1$

$$A - 1 \cdot I = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$ has only one free variable.

Hence A is not diagonalizable.

Turn over →

(c) We need to find λ such that

$$|A + I - \lambda I| = 0$$

i.e $|A - (\lambda - 1) I| = 0$

by (a) $\lambda - 1 = 1, 1, 5$

$\therefore \underline{\underline{\lambda = 2, 2, 6}}.$