

Quiz 4

Name: Solutions.

$$\textcircled{1} \text{ Let } A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

(a) Describe $\text{Nul}(A)$ (b) Describe $\text{Col}(A)$ (c) Give a basis for $\text{Nul}(A)$ and $\text{Col}(A)$ (d) Give another basis for $\text{Col}(A)$.(a) we need to solve $A\vec{x} = \vec{0}$.

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \begin{array}{l} R_1' = R_2 \\ R_2' = R_1 \end{array} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \begin{array}{l} R_2' = R_2 + 3R_1 \\ R_3' = R_3 - 2R_1 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 5 & 10 & -10 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix} \begin{array}{l} R_3' = R_2 \\ R_2' = R_3 \end{array} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 5 & 10 & -10 \end{bmatrix} \begin{array}{l} R_3' = R_3 - 5R_2 \\ \end{array} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $x_4 = k$, $x_5 = l$, ~~$x_2 = m$~~

$$\text{then } x_3 = -2x_4 + 2x_5 = -2k + 2l.$$

Let $x_2 = m$

$$\begin{aligned} \therefore x_1 &= 2x_2 - 2x_3 - 3x_4 + x_5 \\ &= 2m - 2(-2k + 2l) - 3k + l \\ &= 2m + k - 3l \end{aligned}$$

$$\therefore X = \begin{pmatrix} 2m + k - 3l \\ m \\ -2k + 2l \\ k \\ l \end{pmatrix} = m \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix} + l \begin{pmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore \text{Nul}(A) = \text{Span} \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

$$(b) \text{ Col}(A) = \text{span} \left\{ \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} -7 \\ -1 \\ -4 \end{pmatrix} \right\}$$

$$(c) \text{ A Basis for Nul } A \text{ is } \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 2 \\ 1 \end{pmatrix} \right\}$$

A basis for Col A is $\left\{ \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right\}$ since col 1, 3 are pivot columns.

$$(d) \text{ Another basis for Col } A \text{ is } \left\{ \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix} \right\}$$