Quiz 3 Name: Solutions.

① Solve the system using Cramer's rule

$$x_1 + 5x_2 = 0$$
$$2x_1 + 4x_2 - x_3 = 1$$
$$-2x_2 = 0$$

$$\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$A\vec{x} = \vec{b}$: ; $|A| = -(-1)(-2) = -2 \neq 0$

Hence By Cramer's Rule:

$$x_1 = \begin{vmatrix} 0 & 5 & 0 \\ 1 & 4 & -1 \\ 0 & -2 & 0 \end{vmatrix} \Big/ |A| = -1(0)/-2 = 0$$

$$x_2 = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & -1 \\ 0 & 0 & 0 \end{vmatrix} \Big/ |A| = 0/-2 = 0$$

$$x_3 = \begin{vmatrix} 1 & 5 & 0 \\ 2 & 4 & 1 \\ 0 & -2 & 0 \end{vmatrix} \Big/ |A| = -1(-2)/-2 = -1$$

Hence $\vec{x} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$.

(2) Let A, B be $n \times n$ matrices, such that $|A| = 10$, $|B| = 5$

Find $\begin{vmatrix} A_{n\times n} & O_{n\times n} \\ O_{n\times n} & B_{n\times n} \end{vmatrix}$

Note that the given matrix might not be in an echelon form. But if we do row operations on the first n rows and then on the last n rows, we can turn it into ~~an~~ a triangular form where A turns to its echelon form, and B turns into its. Then the product of diagonal entries is the product of diagonal entries of ~~A~~ the echelon form of A and B.

Hence the required determinant is $|A| \cdot |B| = 10 \times 5 = 50$.