Quiz 2 Name: Solutions.

1. Let $A=\begin{pmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{pmatrix}$

(i) Find A^{-1} (if possible)

(ii) If A is not invertible, then find a dependency equation involving the column vectors of A.

(i) $\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ -1 & 5 & 6 & 0 & 1 & 0 \\ 5 & -4 & 5 & 0 & 0 & 1 \end{array}\right] \underset{R_3' = R_3 - 5R_1}{\overset{R_2' = R_2 + R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & 6 & 10 & -5 & 0 & 1 \end{array}\right]$

$\overset{R_3' = R_3 - 2R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & & & \\ 0 & 3 & 5 & & & \\ 0 & 0 & 0 & & & \end{array}\right]$. we have less than 3 pivots. Hence A^{-1} does not exist.

(ii) Since A^{-1} does not exist; the columns of A must be linearly dependent. To find a dependency equation, we solve $A\vec{x} = \vec{0}$

$\begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 3 & 5 \\ 0 & 6 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \left.\begin{array}{l} 3x_2 + 5x_3 = 0 \\ x_1 - 2x_2 - x_3 = 0. \end{array}\right\}$ Let $x_3 = k$ be free

$\therefore x_2 = -\frac{5k}{3}$

$x_1 = 2x_2 + x_3 = \frac{-7k}{3}.$

Let us take any $k \neq 0$, say $k = 1$

$\therefore -\frac{7}{3}\begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} - \frac{5}{3}\begin{pmatrix} -2 \\ 5 \\ -4 \end{pmatrix} + 1\begin{pmatrix} -1 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$ ← This is a dependency equation.