Name: Solutions Quiz 1

(1) Let $\vec{a_1} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\vec{a_2} = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$, $\vec{a_3} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix}$.

Determine if $\vec{b}$ is in Span$\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$.

Sol: Form $[A \,\vdots\, b]$

$$\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 2 & 6 & 3 & -4 \end{bmatrix} \underset{\sim}{R_3' = R_3 - 2R_1} \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & 11 & -12 \end{bmatrix} \underset{\sim}{R_3' = R_3 - 2R_2}$$

$$\left[\begin{array}{ccc:c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & 15 & -14 \end{array}\right].$$

This is in row echelon form and clearly has a solution. Hence $\vec{b}$ is in Span$\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$

(2) State True / False and justify.

(i) If S is a set of linearly independent vectors in $\mathbb{R}^m$, then $|S| \le m$

(ii) Is the converse of the above Statement true ?

(i) True; if not, then we form a matrix A with columns as the vectors in S. Then it has more columns then rows. Hence while solving $A\vec{x} = \vec{0}$, will have at least one free variable and thus linearly dependent, a contradiction.

(ii) No. Consider the set $S = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right\}$

Here $m = 3$ and $|S| = 3$, but its not a linearly independent set since $1 \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 1 \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$