HW 7 Solutions.

4.1

(5) $S = \{ at^2 \mid a \in \mathbb{R} \}$ is a subspace of $\mathbb{P}_n$ because

$S = \text{span}\{t^2\}$

(6) $S = \{ a + t^2 \mid a \in \mathbb{R} \}$ is not a subspace of $\mathbb{P}_n$ because

$0 \notin S$ because ~~even if a~~ $a + t^2 \neq 0$ even if $a = 0$. because coefficient of t^2 is always 1.

(8) $S = \{$ all polynomials $p(x)$ such that $p(0) = 0 \}$; that is all polynomials with 0 as a root, that is every such polynomial is of the form $p(x) = x \cdot p_1(x)$, where $p_1(x)$ is any polynomial of degree one smaller than $p(x)$.

$\therefore S = \{ a_1 t + a_2 t^2 + \cdots + a_n t^n \} = \text{span}\{ t, t^2, \cdots, t^n \}$

Hence S is a subspace.

Or you can also argue by ~~say~~ proving that the 'three' conditions hold in S

15 $W = \left\{ \begin{pmatrix} 3a+b \\ 4 \\ a-5b \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is not a subspace of $\mathbb{R}^3$ because no matter what a, b we pick $\begin{pmatrix} 3a+b \\ 4 \\ a-5b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

16 $W = \left\{ \begin{pmatrix} a-1 \\ a-6b \\ 2b+a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is not a subspace of $\mathbb{R}^3$ because.

for $\vec{0}$ to be in W we must have

$$\begin{matrix} a-1=0 \\ a-6b=0 \\ 2b+a=0 \end{matrix} \quad \text{that} \quad \begin{matrix} a=1 \\ a=6b \\ a=-2b \end{matrix}$$

which is an inconsistent system.

(17) $W=\left\{\begin{pmatrix} a-b \\ b-c \\ c-a \\ b \end{pmatrix} : a,b,c \in \mathbb{R}\right\}$ is a subspace of $\mathbb{R}^4$ because

$$\cancel{W=}\begin{pmatrix} a-b \\ b-c \\ c-a \\ b \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + b\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

i.e. $W = \text{Span}\left\{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}\right\}$.

4.2

20 $A=\begin{bmatrix} 1 & -3 & 9 & 0 & -5 \end{bmatrix}$

Nul A is a subspace of $\mathbb{R}^5$.

Col A is a subsace of $\mathbb{R}$.

24 $A=\begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$, $w=\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. To find if $w \in \text{Col } A$.

check if $A\vec{x}=\vec{w}$ has a solution.

To check if $\vec{w} \in \text{Nul}(A)$ check if $A\vec{w}=\vec{0}$.

(check these yourself).