

HW6 Solutions

3.3

11

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

The cofactors are

$$c_{11} = (+1) \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0 \quad c_{21} = (-1) \begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = 1$$

$$c_{12} = (-1) \begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix} = -3 \quad c_{22} = (+1) \begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} = -1$$

$$c_{13} = (+1) \begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix} = 3 \quad c_{23} = (-1) \begin{vmatrix} 0 & -2 \\ -1 & 1 \end{vmatrix} = 2$$

$$c_{31} = (+1) \begin{vmatrix} -2 & -1 \\ 0 & 0 \end{vmatrix} = 0, \quad c_{32} = (-1) \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = -3$$

$$c_{33} = (+1) \begin{vmatrix} 0 & -2 \\ 3 & 0 \end{vmatrix} = 6.$$

$$\therefore \text{adj}(A) = C^T$$

$$= \begin{bmatrix} 0 & -3 & 3 \\ 1 & -1 & 2 \\ 0 & -3 & 6 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -1 & -3 \\ 3 & 2 & 6 \end{bmatrix}$$

$$|A| = (-1)(3) \begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = -3(-1) = 3$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}(A) = \begin{bmatrix} 0 & 1/3 & 0 \\ -1 & -1/3 & -1 \\ 1 & 2/3 & 2 \end{bmatrix}.$$