

HW 5 Solutions

3.1

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$$\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix} = (-1) \cdot 3 \begin{vmatrix} 3 & 2 & 4 & 0 \\ 0 & -4 & 1 & 0 \\ 5 & 6 & 7 & 1 \\ 2 & 3 & 2 & 0 \end{vmatrix} = -3(-1) \begin{vmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= 3 \left(-4 \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} \right) = 3(-4 \cdot -2 - 1 \cdot 5) = 3(8 - 5) = 9.$$

3.2

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$$\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 1 & 5 & 5 \\ 0 & 2 & 7 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -3 & -5 \end{vmatrix} = - \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= - (1)(1)(-3)(-1) = -3.$$

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$$\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} \xrightarrow{R_2' = R_2 - R_1} \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2 \cdot 7 = 14.$$

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we know that $A \cdot A^{-1} = I$

$$\therefore |A \cdot A^{-1}| = |I|$$

$$\Rightarrow |A| \cdot |A^{-1}| = 1 \Rightarrow |A^{-1}| = \frac{1}{|A|}$$

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rA is the matrix obtained by scaling every entry in A by r . which means scaling every row by r .

$$\text{Hence } |rA| = \underbrace{r \cdot r \cdots r}_{n \text{ times}} |A| = r^n |A|.$$