HW 4 Solutions

2.2 (11) Given that A^{-1} exists.

Now if $AX = B$

$\Rightarrow A^{-1}(AX) = A^{-1}B$ [Multiplying by A^{-1} on the left. On both sides of the equation]

$\Rightarrow (A^{-1}A)X = A^{-1}B$
[by associativity]

$\Rightarrow IX = A^{-1}B \Rightarrow \underline{X = A^{-1}B}$.

Hence the solution is unique and is $A^{-1}B$.

(13) $AB = AC$

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$ [$\because A^{-1}$ exists. and so we multiply by A^{-1} on the left]

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C$ [by associativity]

$\Rightarrow IB = IC \Rightarrow B = C$.

Not true in general if A is not invertible.

example: let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ← Note A^{-1} does not exist.

$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ But $AB = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = AC$

$C = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$ but $B \neq C$.

(21) Consider the system $A\vec{x} = \vec{0}$

~~Pre~~ Multiply by A^{-1} on the left $A^{-1}(A\vec{x}) = A^{-1}\vec{0}$

$$\Rightarrow (A^{-1}A)\vec{x} = \vec{0}$$

$$\Rightarrow I\vec{x} = \vec{0}$$

$$\Rightarrow \vec{x} = \vec{0}$$

Hence $\vec{x} = \vec{0}$ is the only solution. for $A\vec{x} = 0$

Hence columns of A are linearly independent.

(33)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{R_3' = R_3 - R_1}{\overset{R_2' = R_2 - R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\overset{R_3' = R_3 - R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \qquad \therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\substack{R_3' = R_3 - R_2 \\ R_2' = R_2 - R_1}}{\overset{R_4' = R_4 - R_3}{\sim}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

so the form of A^{-1} should be all 1's in the main diagonal and -1's in the diagonal below the main diagonal and 0's everywhere else. Once can check that this actually is the inverse of $A_{n \times n}$.

(2.3)

(2) $A = \begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$. Note $\vec{v}_1 - \frac{2}{3}\vec{v}_2 = \vec{0}$. Hence A is not invertible.

$\begin{array}{cc} \uparrow & \uparrow \\ \vec{v}_1 & \vec{v}_2 \end{array}$

(3) $A = \begin{bmatrix} \boxed{5} & 0 & 0 \\ -3 & \boxed{-7} & 0 \\ 8 & 5 & \boxed{-1} \end{bmatrix}$ is invertible; as it has 3 pivots.

(4) Not invertible, since one column is $\vec{0}$ and hence the set of column vectors is linearly dependent.

(16) No, since then it has less then n pivots and hence the set of column vectors are dependent. Hence A^{-1} would not exist. (IMT !!)

(18) No, since C would have 6 pivots, hence C would be invertible. and $C\vec{x} = \vec{v}$ would have a unique solution $\vec{x} = C^{-1}\vec{v}$. (IMT !!)

(20) E and F would commute i.e $EF = FE$, because by IMT $E = F^{-1}$ and $F = E^{-1}$ and thus

$EF = I$, $FE = I$ $\therefore EF = FE$.