HW1 Solutions

Section 1.1

16

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \overset{R_4' = R_4 + 2R_1}{\sim} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix} \sim$$

$$\overset{R_4' = R_4 - \frac{3}{2}R_2}{\sim} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{bmatrix} \overset{R_4' = R_4 + R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system has a solution and is thus consistent.

18 To find if the three planes intersect, we need to find if the system of linear eq^{ns} has a solution

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

The last equation says $0 = -5$ which is a contradiction. Hence the system is inconsistent.

20 $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{bmatrix}$

The last equation reads $(4+2h)\, x_2 = 0$.

If $h = -2$, then x_2 can be any arbitrary number.

and If $h \neq -2$, then $x_2 = 0$

Thus system is consistent for any value of h.

24 (a) True. Scaling a row by a constant just scales the corresponding equation. Interchanging two rows just swaps two equations in the system. Replacement just replaces an equation by another equation which is the sum of itself and a multiple of another. All these operations clearly do not affect the solution set

(b) False. Consider the systems

$$\underbrace{\begin{matrix} x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \end{matrix}}_{\downarrow} \quad \text{and} \quad \underbrace{\begin{matrix} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \end{matrix}}_{\downarrow}$$

$(\frac{1}{2}, \frac{1}{2})$ is the only solution

$(1, 0)$ is the only solution.

Hence they are not row equivalent even though they have equal # rows.

(c) False, by definition an inconsistent system has no solutions.

(d) True. by definition of equivalent systems.

Section 1.2

(6) $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \blacksquare & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(10) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \overset{R_2' = R_2 - 3R_1}{\sim} \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{bmatrix} \sim$

$\overset{R_1' = R_1 + R_2}{\sim} \begin{bmatrix} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

$\therefore \; x_1 - 2x_2 = -4$

$x_3 = -7$

Hence $x_3 = -7$

$x_2 = \text{free} = k$ (say)

$x_1 = 2x_2 - 4 = 2k - 4$

$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2k-4 \\ k \\ -7 \end{pmatrix}$