HW 11 Solutions

6.1

19 (a) True ; by definition of $\|v\|$

(b) True $\quad u\cdot(cv) = \sum_{i=1}^{n} u_i(cv_i) = c\sum_{i=1}^{n} u_i v_i$

$= c(u\cdot v)$

(c) True : if $\|u-v\| = \|u-(-v)\|$

then $\|u-v\|^2 = \|u+v\|^2$

$\Rightarrow (u-v)\cdot(u-v) = (u+v)\cdot(u+v)$

$\Rightarrow u\cdot u - v\cdot u - u\cdot v + v\cdot v = u\cdot u + v\cdot u + u\cdot v + v\cdot v$

$\Rightarrow 4(u\cdot v) = 0$

$\Rightarrow u\cdot v = 0$

$\Rightarrow$ u and v are orthogonal.

(d) False ; $(\text{Col } A)^{\perp} = \text{Nul}(A^T)$ or give a counter-example.

(e) True ; since any vector in W is of the form

$c_1\vec{v}_1 + \cdots + c_p\vec{v}_p$

and if $\vec{x}\cdot\vec{v}_i = 0$ for every i

then $\vec{x}\cdot(c_1\vec{v}_1 + \cdots + c_p\vec{v}_p) = c_1(\vec{x}\cdot\vec{v}_1) + \cdots + c_p(\vec{x}\cdot\vec{v}_p)$

$= 0 + 0 + \cdots + 0$

$= 0$

Hence $\vec{x} \in W^{\perp}$

6.2 (13) $\vec{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $u = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$

$$\hat{y} = \left(\frac{\vec{y}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \left(\frac{8-21}{16+49}\right)\vec{u}$$

$$= \left(\frac{-13}{65}\right)\begin{pmatrix} 4 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} -4/5 \\ 7/5 \end{pmatrix}$$

& $\vec{z} = \vec{y} - \hat{y}$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -4/5 \\ 7/5 \end{pmatrix} = \begin{pmatrix} 14/5 \\ 8/5 \end{pmatrix}$$

$$\therefore \vec{y} = \begin{pmatrix} -4/5 \\ 7/5 \end{pmatrix} + \begin{pmatrix} 14/5 \\ 8/5 \end{pmatrix}$$

6.3 (21) (a) True ; same argument as in 6.1, 19(e)

(b) True, by Orthogonal decomposition theorem

(c) False, Note that in the Orthogonal decomposition theorem's first line ~~its [illegible]~~ says that every $\vec{y}$ can be expressed $\boxed{\text{uniquely}}$ as $\hat{y} + \vec{z}$ where $\hat{y} \in W$ and $\vec{z} \in W^{\perp}$. It does not depend on the orthogonal basis of W.

(d) True ; ~~if $y \in W$ then $\hat{y} = \vec{y}$~~

Since $\vec{y}$ can be expressed as $\vec{y} = \vec{y} + \vec{0}$ and hence by Orthogonal decomposition theorem $\hat{y} = \vec{y}$