

1(a). Prove the following identity for determinants.

$$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 + e_1 \\ a_2 & b_2 & c_2 & d_2 + e_2 \\ a_3 & b_3 & c_3 & d_3 + e_3 \\ a_4 & b_4 & c_4 & d_4 + e_4 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 & e_1 \\ a_2 & b_2 & c_2 & e_2 \\ a_3 & b_3 & c_3 & e_3 \\ a_4 & b_4 & c_4 & e_4 \end{vmatrix}$$

1(b). Use 1(a) to show that

$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + acd \\ 1 & c & c^2 & c^3 + abd \\ 1 & d & d^2 & d^3 + abc \end{vmatrix} = 0$$

(Hint: Remember that column operations are also allowed here.)