MATH 582, SPRING 2019 – PROBLEM SET 2

Do five of the six problems below. Due Wednesday, February 27.

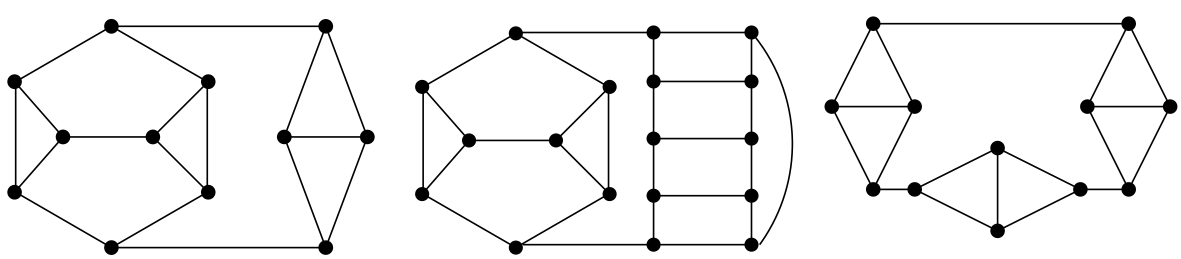
1. Prove the following statements about graceful graphs:
(a) A graceful graph may have a non-graceful component.
(b) A graph with all components graceful need not be graceful.
(c) Every graceful forest is a tree.

2. Prove that if the Graceful Tree Conjecture is true and T is a tree with m edges, then K_{2m} can be decomposed into $2m-1$ copies of T. (Hint: Use the proof of Theorem 6.1.41 for a suitable tree with $m-1$ edges.)

3. A partial case of the Corradi-Hajnal Theorem (which is also a partial case of Hajnal-Szemerédi Theorem which in turn is a partial case of the Bollobás-Eldridge-Catlin Conjecture) says that for every positive integer s, each $3s$-vertex graph G with minimum degree at least $2s$ contains a spanning subgraph all whose components are triangles.
(a) Using this fact, prove that for every positive integer s, each $(3s-1)$-vertex graph G with minimum degree at least $2s-1$ contains a spanning subgraph with one component K_2 and all other components being triangles.
(b) Using the above, prove that for every integer $s \geq 4$, each $(3s-1)$-vertex graph G with minimum degree at least $2s-1$ contains a spanning subgraph with two components $K_4 - e$ and all other components being triangles.

4. For positive integers n and m, let $t(n,m)$ be the minimum number such that every n-vertex graph with at least $1+t(n,m)$ edges contains every m-edge graph with no isolated vertices. For $n \geq 1000$, find $t(n,2)$ and $t(n,3)$. (Hint: You can use Theorem 6.1.6.)

5. For each of the three graphs below, determine whether it has a nearly equitable 3-coloring or not. If it has, show it. If it hasn't, prove it.



(a) **(b)** **(c)**

6. Prove that the vertex set of every 4-degenerate graph G with maximum degree 5 can be partitioned into sets V_1, V_2, and V_3 so that V_1 is independent, and every component of $G(V_2)$ and of $G(V_3)$ is a path (possibly, with just one vertex). (Hint: Prove first that you can partition $V(G)$ so that V_2 and V_3 do not induce cycles, i.e. $G[V_2]$ and $G[V_3]$ are 1-degenerate. Compare with Exercise 6.2.54.)