1. Prove that every graph with maximum degree at most 3 is totally-5-colorable using the following plan. Suppose $G$ is a minimum counter-example.
   a) Prove that $G$ has no cut edges;
   b) Prove that $G$ is 3-regular;
   c) Conclude that $E(G)$ has a perfect matching $M$ and color $M$ with Color 5;
   d) Color everything else with 1, 2, 3, 4: First, color the vertices and then use list edge coloring.

2. Let $G$ be a graph, $A \subset V(G)$ be an independent set in $G$ and $B = V(G) - A$. Let $G'_A$ be the digraph obtained from $G$ by orienting the edges connecting $A$ with $B$ arbitrarily and replacing each edge $xy \in E(G[B])$ with the pair $\{xy, yx\}$ of opposite directed edges. Prove that $G'_A$ is kernel-perfect. (Comment: This is a generalization of Richardson’s Theorem on orientations of bipartite graphs.) (Hint: Modify the proof of Richardson’s Theorem given in class.)

3. Problem 3.4.11 in the book.


5. Prove that every simple plane 3-connected graph has either a 3-vertex incident with a face of length at most 5 or a 3-face incident with a vertex of degree at most 5.

6. Problem 3.4.31 in the book. (Hint: For arbitrary edges $A$ and $B$ in $H$, bound the probability that in a random permutation the last vertex of $A$ is the first vertex of $B$.)