MATH 581, FALL 2015 – PROBLEM SET 1

Do five of the six problems below. Due Friday, September 25.

1. Problem 2.1.33 in the book.
2. Problem 2.1.36 in the book.
3. Problem 2.1.48 in the book. Hint: Use a bigraph model \( H = (X, Y; E) \) and construct an auxiliary bigraph \( H' = (X', Y'; E') \) such that the existence of a matching in \( H' \) covering \( X' \) is equivalent to the original claim.
4. Let \( G \) be an \( n \)-vertex 3-regular graph with at most 5 cut-edges. Prove that \( G \) has a matching with at least \( 0.5n - 1 \) edges. What about 6 cut-edges?
5. Problem 2.1.91 in the book.
6. Prove that in the stable matching problem, among all stable matchings, (a) every man has the happiest marriage in the matching produced by the Gale-Shapley Proposal Algorithm with men proposing, and (b) every woman has the unhappiest marriage in the matching produced by this algorithm (with men proposing).