Math 412  

HW7

Due Wednesday, November 07, 2018

Solve four of the next five problems.

1. Prove that for every simple graph $G$ with maximum degree at most 3, $\kappa'(G) = \kappa(G)$.

2. Let $G$ be a 2-connected simple graph.
   (a) Prove that in every ear decomposition of $G$, the number of ears (including the initial cycle) is $|E(G)| - |V(G)| + 1$.
   (b) Let $s$ and $t$ be vertices in this 2-connected graph $G$. Prove that the vertices of $G$ can be linearly ordered so that each vertex apart from $s$ and $t$ has a neighbor that is earlier in the order and a neighbor that is later in the order.

3. Prove that a simple 2-connected graph $G$ with at least four vertices is 3-connected if and only if for every triple $(x, y, z)$ of distinct vertices and any edge $e$ not incident with $y$, $G$ has an $x, z$-path through $e$ that does not contain $y$.

4. Let $k \geq 3$. Prove that for every $n \geq k + 1$, every $k$-connected $n$-vertex graph $G$, and every disjoint vertex sets $S$ and $T$ in $G$ with $|T| = 3$ and $|S| = k - 3$, there is a cycle that contains $T$ and is disjoint from $S$. Give an example of a 2-connected graph and some 3 vertices in this graph that do not belong to a common cycle.

5. Let $xy$ be an edge in a digraph $G$. Prove that $\kappa(G - xy) \geq \kappa(G) - 1$.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 4.1: # 1, 2, 3, 5, 7. Section 4.2: # 1, 2, 3, 4, 6. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 4.1: # 10, 12, 14, 24, 31.
Section 4.2: # 8, 9, 11, 12, 20, 21, 23, 24, 26. Do not write these up!