Math 412          HW5

Due Wednesday, October 10, 2018

Solve four of the next five problems.

1. Use Prim’s Algorithm and Kruskal’s Algorithm to find minimum spanning trees in
the weighted graph on the other side of this sheet (show the sequences in which edges are
added).

2. Find the number of spanning trees in the graph obtained from \( K_n \) by adding an edge.
(You can use Cayley’s Formula and the result of Problem 2.2.7 in the book.)

3. Let \( G \) be a connected regular bipartite simple graph. Let \( G' \) be obtained from \( G \) by
deleting one vertex from each of the two partite sets. Prove that \( G' \) has a perfect matching.

4. A standard card deck has 52 cards with 4 suits and 13 values. The cards are dealt
into an array with 4 rows and 13 columns.
(a) Prove that there is a set of 13 cards, one in each column, having distinct values.
(b) Using (a), prove that by a sequence of exchanges of cards with the same value, one can
rearrange the cards so that each column contains exactly one spade.

5. # 3.1.28 in the book.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 2.2: # 1, 2, 3. Section 3.1: # 1, 2, 3, 4, 6. Do not
write these up!

OTHER INTERESTING PROBLEMS: Section 2.2: # 5, 7, 8, 17. Section 3.1: # 8, 9,
10, 21, 24, 25, 30.
Do not write these up!