Math 412        HW2

Due Wednesday, September 12, 2018

Solve four of the next five problems.

1. Prove or disprove:
   (a) Every connected graph $G$ has a closed walk that traverses each edge of $G$ exactly two times;
   (b) Every connected graph $G$ has a closed walk that traverses each edge of $G$ exactly three times.

2. For $k \geq 2$, prove that every $k$-regular bipartite graph has no cut-edge and construct a bipartite graph with all vertex degrees in $\{k, k+1\}$ that has a cut-edge.

3. Given an integer $k \geq 2$, let $G(k)$ be the subgraph of the cube $Q_{2k}$ induced by the vertices in which the number of ones is either $k-1$ or $k$. Compute the number of vertices, the number of edges, and the girth (the length of a shortest cycle) of $G(k)$. What are the degrees of the vertices in $G(k)$?

4. (a) Prove that for $n \geq 3$, every connected $n$-vertex graph contains a maximum independent set that contains all leaves (vertices of degree 1);
   (b) Give an example of a connected graph $G$ with 4 leaves and a maximum independent set $I$ in $G$ that contains only one leaf.

5. Extending the proof of Mantel’s Theorem given in class (see handout on the class webpage), prove that for each $n \geq 1$, every $n$-vertex triangle-free simple graph with the maximum number of edges is isomorphic to $K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}$. (Other proofs do not count.)

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 1.2: # 1, 4, 5, 8, 9, 10, 11. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 1.2: # 14, 18, 20, 23, 41.
Section 1.3: # 17. Do not write these up!