Math 303, Section D1 - Test #1 - February 8, 2002

Answer all questions. No books or notes allowed. A calculator may be used. 55 minutes. 100 points total.

1. Give the definition of each of the following:

   (a) (5 points) Parallelogram.

   (b) (5 points) Centroid of the set of points \( \{ A_1, A_2, \ldots A_n \} \).

   (c) (5 points) Barycentric coordinates of a point \( P \) with respect to (non-collinear) points \( A, B, C \).
2. (10 points) Prove that the centroid of a triangle is also the centroid of the triangle of the midpoints of its sides.
3. (10 points) Find the centroid of the mass points $(2, A), (3, B), (1, C), (-2, D)$. 
4. (15 points) Lines \( l_{AB} \) and \( l_{CD} \) are defined to be parallel if there is some number \( t \) such that \( A - B = t(C - D) \). Prove that parallel lines do not intersect (hint: proof by contradiction).
5. (10 points) Give the statement of Menelaus’ theorem.

6. (10 points) Give the statement of Ceva’s theorem.
7. (15 points) Use Ceva’s theorem to prove that the medians of a triangle must be concurrent.
8. (8 points) Ceva’s theorem is an “if and only if” statement. Prove one of the directions of the theorem (your choice). Be sure to state clearly which direction you are proving.