7.3.1 In the Klein model, two lines are right-limiting parallels to each other if and only if they approach the same boundary point. So if $l$ is right-limiting to $m$, then $m$ is right-limiting to $l$.

7.3.2 If $l$ is right limiting to $m$ and $n$ is right limiting to $n$, then all three lines approach the same boundary point. Therefore $l$ is also right-limiting to $n$.

7.3.3 Let $m$ be right limiting to $l$. Then $r_\pi(l) = l$ and $r_\pi(m)$ is another line, which we denote by $n$. Since reflection preserves parallelism, $n$ is parallel to $l$. To show that $n$ is a right-limiting parallel to $l$, let $Q$ be a point on $n$. Drop a parallel-freepenpendicular from $Q$ to a point $R$ on $l$. Let $P = r_\pi(Q)$. Then $Q$ is on $m$ and $QR \perp l$ (reflection preserves angles).
Let $l$ be a line within angle $LRQT$. Then $r_\ell(t)$ is a line within angle $LRPS$. Since $m$ is right-limiting to $l$, $r_\ell(t)$ intersects $l$. Therefore $t$ intersects $l$ (since a reflection sends intersecting lines to intersecting lines).

Let $w$ be a line outside $LRQT$. Then $r_\ell(w)$ is outside $LRPS$. Since $m$ is right-limiting to $l$, $r_\ell(w)$ does not intersect $l$. Therefore $w$ does not intersect $l$ (since a reflection sends parallel lines to parallel lines).

So $n$ separates lines which intersect $l$ from those which do not, so $n$ is a right-limiting parallel to $l$.

An $o$ The right omega point of $l$ is the set of all right-limiting parallels to $l$. The first part shows that $r_\ell$ maps each of these limiting parallels to
7.3.3 cont'd

another limiting parallel, so it maps the set of right-limiting parallels of \( l \) to itself. This means it fixes the right omega point of \( l \).

7.3.11 Let \( \overline{PQ} \), \( l \), \( \overline{R} \) be as described.

Let \( \overline{PQ'} \) also have length \( h \), let \( l' \) be \( \perp \) to \( \overline{PQ'} \) at \( Q' \), let \( \overrightarrow{P'R'} \) be the limiting parallel to \( l' \) at \( P' \).

This defines two omega triangles \( \triangle PQR \) and \( \triangle P'R'Q' \).

Since \( \overline{PQ} \cong \overline{P'Q'} \) and \( \angle PQS = \angle P'Q'S' = 90^\circ \),

Theorem 7.8 \( \Rightarrow \angle QPS = \angle Q'S'P'S' \).

This is the angle \( \alpha(h) \). It is well defined because it depends only on \( h \), not on the particular segment \( \overline{PQ} \) which is chosen.

7.3.12 Let \( \overline{PQ'} \) have length \( h' \) and find \( Q \) on \( \overline{PQ'} \) so \( \overline{PQ} \) has length \( h \).

Construct \( l \) and limiting parallels \( \overrightarrow{Q'R} \), \( \overrightarrow{Q'R'} \) to \( l \).

Then \( \overrightarrow{QQ'}, \overrightarrow{QR}, \overrightarrow{Q'R'} \) form an omega \( A \). By Exterior \( \perp \) Theorem 7.7, \( \alpha(h) > \alpha(h') \).
Geom Explorer Problems.

1. If you make a large Saccheri quad., you get the summit angles as close as you like to $0^\circ$. My picture is poor!

2. If you make a very small Saccheri quad., you get the summit angles as close as you like to $90^\circ$.

Note: Summit angles never = $0^\circ$ or $90^\circ$

$0^\circ < \text{summit angle} < 90^\circ$

3. Not such a great picture!

4. From mathworld.wolfram.com:

\[ a(h) = 2 \tan^{-1}(e^{-h}) \]

Notice $\lim_{h \to 0} a(h) = 90^\circ$, $\lim_{h \to 0} a(h) = 0^\circ$, the function $a(h)$. 

Ok, I just a chart is